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# Relativistic self-focusing of an intense laser beam in an inhomogeneous plasma

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**Abstract** A steady state analysis of self focusing of a Gaussian laser beam in linearly increasing inhomogenous plasma due to non-linearity produced by relativistic effect has been presented The intensity dependent non-linear part of the dielectric constant has been chosen to be arbitrarily large Paraxial-ray approach of Akhmanov and Sodha has been considered during the analysis Due to inhomogeneity, focusing appears to be prominent and found to depend upon characteristic length of inhomogeneity Two values of critical power obtained are found not to be dependent on inhomogeneity

**Keywords** Self-focusing of beam, Laser-plasma interaction, non-linear phenomena

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## 1. Introduction

The problem of interaction of an intense laser beam with plasma is of current interest because of the possibility of heating plasmas to high temperatures and achieving laser controlled fusion [1,2]. The rapid recent advances in lasers are making it possible to achieve fields (laser energy density $10^{16} - 10^{18}$ W/cm$^2$) in which plasma electron oscillates at relativistic velocities. The increased effective mass of the electron due to relativistic effect reduces the electron plasma frequency and allows laser beam to penetrate into the region with density above critical level [3]. It is worth while to develop systematic analysis of this relativistic effect and incorporate it in the study of self-focusing, a work already undertaken in our laboratory [4,5]. The fact that the relativistic mechanism is the only mechanism of self-focusing that can manifest itself for sub-picosecond pulses makes the understanding of this mechanism very important [6, 7]. Recently, a new generation of compact lasers has been developed, capable of providing power exceeding 1T in short pulses whose width is one picosecond or less [8]. After focusing, the pulse irradiance can reach $10^{18}$ W/cm$^2$ and give the free electrons of a plasma a relativistic motion which induces the so called relativistic self-focusing of laser beam. This mechanism is of great importance since it could produce ultrahigh laser irradiance exceeding $10^{19} - 10^{20}$

W/cm$^2$ over distances much greater than the Rayleigh length determined by natural diffraction. As a result, it would favour the observation of new plasma effects such as high order harmonic generation by relativistic electrons [9] frequency up shifting [10], high energy electron acceleration [11] etc. The earlier analysis of relativistic self-focusing of laser beam of plasma by Siegrist [12], was essentially a perturbation treatment, based on quadratic dependence of the dielectric constant on the electric field of the beam. Such analysis has limited applicability to the understanding of self-focusing of beams with arbitrarily large electric field and consequently arbitrarily large non-linear part of the dielectric constant.

In this paper, we present an analysis of self-focusing of a Gaussian laser beam of arbitrary large intensity in an inhomogeneous plasma due to non-linearity produced by relativistic effect only. The steady state paraxial theory, as developed by Akhamanov *et al* [13,14], is employed here.

## 2. Basic theory of non-linear dielectric constant and self-focusing

The relativistic self-focusing [15] of an intense laser beam (energy density of the order of $10^{16}$–$10^{18}$ W/cm$^2$) in plasma is based on dependence of optical constant (refractive index) on the relativistic change of the mass of the oscillating electron.

Under the influence of propagating intense laser beam electric field, electrons acquire large oscillatory velocity and hence the effective refractive index of the plasma is given by

$$\check{n}^2 = 1 - \frac{\omega_p^2}{\omega^2}\frac{m}{m_r} \tag{1}$$

The plasma frequency in the absence of beam is

$$\omega^2_p = \frac{4\pi n e^2}{m},$$

where $n$ is the plasma density, $m$ is the rest mass and $m_r$ the relativistic mass of the electron.

The equation of motion of an electron in the oscillating electric field of laser beam

$$E = E_0(r)\, e^{-i\omega t}$$

can be written as

$$\frac{dp}{dt} = -e\,E_0\,(r)\,e^{-i\omega t} \tag{2}$$

where $p = m_r v$ and $m_r = \dfrac{m}{\sqrt{1 - v^2/c^2}}$.

Thus, the time average of relativistic mass $m_r$ of the electron considering quiver velocity can be written as

$$m_r = m\left[1 + \frac{e^2 E_0 E_0^*}{2m^2\omega^2c^2}\right] \tag{3}$$

The dielectric constant of plasma due to relativistic effect of propagating laser beam can be written as

$$\varepsilon = \check{n}^2 = 1 - \frac{\omega^2{}_p}{\omega^2}\left[1 + \frac{e^2 E_0 E_0^*}{2m^2\omega^2 c^2}\right]^{-1/2} . \tag{4}$$

Now following Sodha *et al* (14), the linear part of the dielectric constant of the medium is

$$\varepsilon_0 = 1 - \frac{\omega^2{}_p}{\omega^2} \tag{5a}$$

and the non-linear part of the dielectric constant of plasma due to relativistic effect of propagating laser beam is

$$\Phi\,(E_0 E^*{}_0) = \frac{\omega^2{}_p}{\omega^2}\left\{1 - \left(1 + \frac{\alpha\, E_0 E^*{}_0}{2}\right)^{1/2}\right\}. \tag{5b}$$

Here $\alpha = \dfrac{e^2}{m^2\omega^2 c^2}$ is a term which depends upon the frequency of laser beam.

In the paraxial – ray approximation $\Phi$ is expanded around $\Phi \approx 0$. However, with such an expansion, one can study only those cases where non-linearity term $\Phi$ is very small as compared to linear term $\varepsilon_0$. In case of arbitrary large non-linearity one should expand $\Phi$ around an arbitrary large value at $r = 0$. In order to do this, the effective dielectric constant of equation (5) may be rewritten as

$$\varepsilon_0 + \Phi\,(< E_0 E_0^* >) = \varepsilon_0 + \Phi\left(\frac{k(0)}{k(f)}\frac{E_0^2}{2f^2}\right) + \Phi\,(< E_0 E_0^* >)$$

$$- \Phi\left(\frac{k(0)}{k(f)}\frac{E_0^2}{2f^2}\right), \tag{6}$$

where $f$ is the dimensionless beam-width parameter and $k$ is the propagation constant defined below as

$$k(f) = \frac{\omega}{c}\,[\varepsilon_0(f)]^{1/2} \quad \text{and} \quad k(0) = \frac{\omega}{c}\,[\varepsilon_0'(f = 1)]^{1/2} .$$

In the eq. (6), < > denote the time average over many cycles.

Using the technique suggested by Maheshwari *et al* [16], one can write, the effective dielectric constant for arbitrary large magnitude of non-linearity as

$$\varepsilon = \varepsilon_0' - \varepsilon_1 r^2 \tag{7}$$

where $\varepsilon'_0$ and $\varepsilon_1$ represent the linear and non-linear dielectric constant for arbitrary magnitude of non-linearity. Here $\varepsilon_1$ is a function of $f(z)$ – the dimensionless beam width parameter.

Considering the propagation of a linearly polarized Gaussian beam with its electric vector polarized along the $z$- axis, at $z = 0$, the intensity distribution of the beam is given by

$$EE^* = E_0^2 \exp\left(-\frac{r^2}{r_0^2}\right), \tag{8}$$

where $r$ is the radial co-ordinate of the cylindrical co-ordinate system and $r_0$ is the initial beam width.

Adopting an approach similar to that of Akhmanov *et al* [13] and Sodha *et al* [17], we obtain the following equation

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2(f)\, r_0^4\, f^3} - \frac{\omega^2 \varepsilon_1(f) f}{c^2 k^2(f)}. \tag{9}$$

The eq. (9) governs the variation of beamwidth parameter ($f$) with distance of propagation ($z$). The first term of RHS corresponds to diffraction divergence of the beam and the second term corresponds to focusing.

## 3. Self-focusing in axially inhomogeneous plasma

The plasma considered in the present study has uniform electron density in the plane perpendicular to the beam propagation direction and electron density variation along $z$- direction is written as

$$n = n_0\, W(z), \tag{10}$$

where $n_0$ is the density of plasma at $z = 0$ and $W(z)$ is the density profile function and is taken as

$$W(z) = \left(1 + \frac{z}{L}\right). \tag{11}$$

Here $L$ is the characteristic scale length of inhomogeneity. This function is plotted in Figure 1.

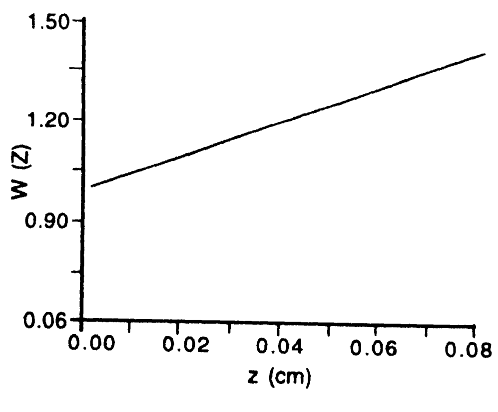


**Figure 1.** Variation of inhomogeneity function with propagation distance ($z$).

For this type of inhomogeneous plasma (linearly increasing electron density along $z$-direction) plasma frequency becomes $z$-dependent and is given by

$$\omega_p^2 = \frac{4\pi e^2}{m} n_0 \left(1+\frac{z}{L}\right)$$

$$= \omega_{p0}^2 \left(1+\frac{z}{L}\right), \tag{12}$$

where $\omega_{p0}$ is the plasma frequency at the boundary ($z = 0$).

Thus for inhomogeneous plasma, the non-linear term $\Phi$ can be expanded [16] for relativistic non-linear effect as

$$\Phi(E E^*) \approx \Phi\left(\frac{k(0)}{k(f)} \frac{E_0^2}{2f^2}\right) - r^2 \left[\frac{\omega_{p0}^2}{\omega^2}\left(1+\frac{z}{L}\right)\frac{k(0)}{k(f)} \frac{\alpha E_0^2}{4r_0^2 f^4} \times \left(1+\frac{k(0)}{k(f)} \frac{\alpha E_0^2}{2f^2}\right)^{-1/2}\right]. \tag{13}$$

Only terms up to $r^2$, are considered here.

In the above equation,

$$\Phi\left(\frac{k(0)}{k(f)} \frac{E_0^2}{2f^2}\right) = \frac{\omega_{p0}^2}{\omega^2}\left(1+\frac{z}{L}\right)\left[1-\left(1+\frac{k(0)}{k(f)} \frac{\alpha E_0^2}{2f^2}\right)^{-1/2}\right]$$

Substituting the value of $\Phi(EE^*)$ in the effective value of dielectric constant

$$\varepsilon = \varepsilon_0 + \Phi(EE^*)$$

and rearranging the equation according to eq. (7), leads to

$$\varepsilon_0' = \varepsilon_0 + \frac{\omega_{p0}^2}{\omega^2}\left(1+\frac{z}{L}\right)\left[1-\left(1+\frac{k(0)}{k(f)} \frac{\alpha E_0^2}{2f^2}\right)^{-1/2}\right], \tag{14a}$$

$$\varepsilon_1 = \frac{\omega_{p0}^2}{\omega^2}\left(1+\frac{z}{L}\right)\frac{k(0)}{k(f)} \frac{\alpha E_0^2}{4r_0^2 f^4}\left(1+\frac{k(0)}{k(f)} \frac{\alpha E_0^2}{2f^2}\right)^{-1/2}. \tag{14b}$$

These values of $\varepsilon_0'$ and $\varepsilon_1$ are incorporated in eq. (9), which gives the equation for the beam width parameter as

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2(f) r_0^4 f^3} - \frac{\omega_{p0}^2}{k^2(f) c^2}\left(1+\frac{z}{L}\right)\frac{k(0)}{k(f)} \frac{\alpha E_0^2}{4r_0^2 f^3}\left(1+\frac{k(0)}{k(f)} \frac{\alpha E_0^2}{2f^2}\right)^{-1/2}. \tag{15}$$

## 4. Result and discussion

The solution of the above equation (15) reveals many important features concerning the self-focusing behaviour of intense laser beam (relativistic) propagation in inhomogeneous plasma.

This equation is solved numerically with the help of computer. For computation a typical set of parameters $\omega_p = 8.5 \times 10^{13}$ rad sec$^{-1}$, $r = 30\,\mu$m, $\omega = 1.7 \times 10^{14}$ rad sec$^{-1}$ ($CO_2$ laser), $\alpha E_0^2 = 1.5$ and $L = .02$ has been chosen.

Beam width parameter $f$ representing self-focusing phenomena, has been plotted as a function of propagation distance $z$, using the same numerical parameters in Figure 2. In this figure, results are also plotted for the homogeneous plasma medium, for comparison.

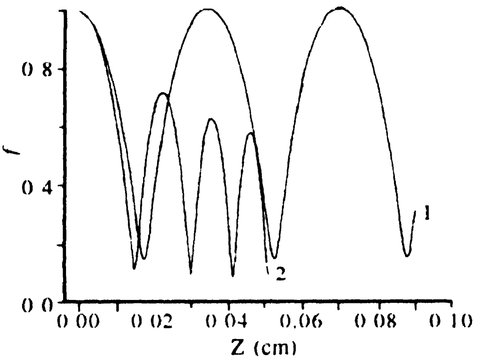


**Figure 2.** Variation of focusing parameter ($f$) with propagation distance ($z$) for homogeneous (1) and axially inhomogeneous (2) plasma of characteristic length ($L$ = 0 02)

It is observed that the beam alternatively converges and diverges as it propagates, for both inhomogeneous as well as homogeneous medium. But in case of inhomogeneous medium, the beam converges quickly and strongly as compared to homogeneous medium.

Because of propagating beam strong field and related relativistic effects, self-focusing channel is formed, which acts as an oscillatory wave-guide. Beam may penetrate into the plasma even with the density above the critical level.

Because of variation in the beam aperture due to self-focusing, intensity of the propagating beam will change in the plasma. Variation of beam intensity $I$ vs $Z$ is shown in Figure 3. In the paraxial part of the beam, the intensity shows sharp peaks. At these points

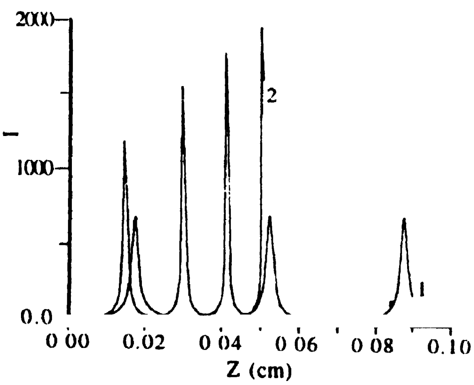


**Figure 3.** Variation of beam intensity ($I$) with propagation distance ($Z$) for homogeneous (1) and axially inhomogeneous (2) plasma.

intensity of the beam is high. As seen from the Figure 3, intensity peaks are very strong in case of inhomogeneous plasma medium. At these peaks, the non-linear refraction forces decreases due to increasing field intensity at the beam axis and diffraction forces becomes dominant, causing the beam to diverse.

Figure 4 shows the dimensionless beam width parameter ($f$) as a function of $z$ for

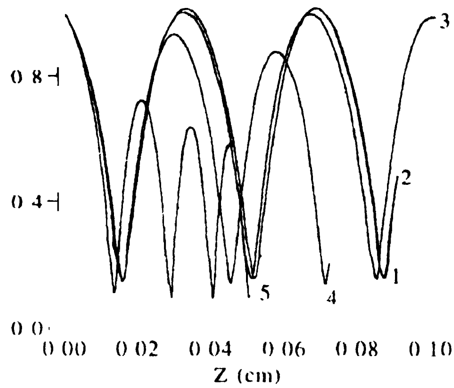


**Figure 4.** Variation of focusing parameter ($f$) with propagation distance for axially inhomogeneous plasma of different characteristics length $L$ (1) $L$ = 200, (2) $L$ = 20, (3) $L$ = 2, (4) $L$ = 0 2, (5) $L$ = 0 02

various values of $L$-the characteristic length of inhomogeneity. For linearly increasing density profile, as considered in the present study, the self-focusing length $Z_f$ decreases with decreasing value of $L$. It appears that the self-focusing in strongly observed for lower value of $L$ as reflected in the numerical value of $f_{min}$ (see Table 1). When the two terms of the RHS of eq. (15) cancel each other and the beam propagates without convergence or divergence, the condition for uniform wave propagation at $z = 0, f = 1$, $\frac{df}{dz} = 0$ is

$$\left(\frac{\omega_p r_0}{c}\right)^2 = \frac{2(1+\alpha E^2_{0r}/2)^{3/2}}{\alpha E^2_{0r}/2}. \tag{16}$$

**Table 1.** Results of self focusing for axially inhomogeneous plasma for the relativistic non-linearity

| S No | Characteristic scale length of inhomogeneity (cm) | Self-focusing length $z_f$ (cm) | | Beam width parameter $f_{min}$ (depth) | |
|---|---|---|---|---|---|
| | L | First self-focusing point | Second self-focusing point | First minima | Second minima |
| 1 | 200 | 0 01769 | 0.05279 | 0 1479 | 0 1477 |
| 2 | 20 | 0.01769 | 0.05249 | 0.1479 | 0.1474 |
| 3 | 2 | 0.01739 | 0.05189 | 0.1473 | 0 1453 |
| 4 | 0.2 | 0.01709 | 0.04649 | 0.1427 | 0.1351 |
| 5 | 0.02 | 0.01499 | 0.03030 | 0.1124 | 0.0982 |

The variation of $\alpha E^2_{0cr}$ *vs* $\frac{\omega_p r_0}{c}$ is shown in Figure 5. For a particular value of $\frac{\omega_p r_0}{c}$, we get two values of $\alpha E^2_{0cr}$. The critical power of the beam has been calculated using the relation

$$P = \frac{c}{8} r^2_0 E^2_{0cr} [\varepsilon'_0 (f = 1)]^{1/2} \tag{17}$$

which gives two critical power $P_{cr1}$ and $P_{cr2}$ corresponding to $E_{0cr1}$ and $E_{0cr2}$ at particular $r_0$.

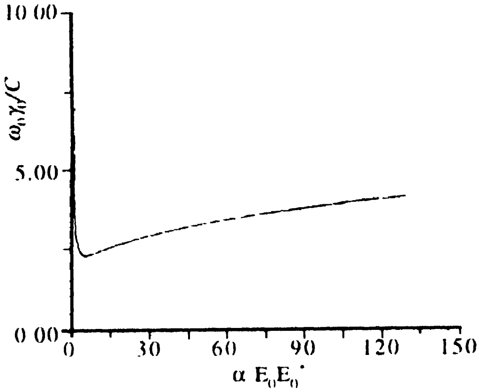


**Figure 5.** Variation of $\left(\frac{\omega_p r_0}{c}\right)$ with $\alpha E_0 E_0^*$

The result indicates that the beam can be self-focused when the power of the beam lies between the two critical values, $P_{cr1} < P < P_{cr2}$ and the range $(P_{cr2} - P_{cr1})$ increases rapidly with increasing $\frac{\omega_p r_0}{c}$. Values of both critical powers come out as $P_{cr1} = 12.3 \times 10^{11}$W and $P_{cr2} = 3.28 \times 10^{14}$W respectively which is nearly same as that calculated by Asthana *et al* [18] for the homogeneous plasma medium. From these results, it appears that the values of critical power are not affected by the inhomogeneity. It may be because in the present study, the plasma is considered to have uniform electron density in the plane perpendicular to the beam propagation direction and the electron density varies linearly along $z$- direction as $(1 + z/L)$. If we apply the conditions to obtain critical power in eq. (15) at $z = 0$ as stated above, the results appear to be same as that of homogeneous plasma because the inhomogeneity term $(1 + z/L)$ reduces to 1. In the paraxial part of the beam, the intensity in the channel which forms after self-focusing is nearly uniform. The dielectric constant of the medium in this region is positive and the jump in density and related effects do not influence the process. In case of radially inhomogeneous plasma, the critical beam power beyond which self-focusing appears, depends strongly on plasma inhomogeneity [19] and suggests the plasma density tailoring in order to lower the critical power.

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